A Design Theory for Wide-Band Parametric Amplifiers

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Abstract—A new exact design theory for a nondegenerate parametric amplifier with double-tuned signal circuit and single-tuned idler circuit is described. If the resistance of the signal circuit, which is neglected in previous papers, is considered, there exists a frequency band in which the amplifier gain is positive. In this paper the band characteristics of the gain are related to this frequency band. Slope parameters of the idler and signal circuits are normalized by the slope parameters which are associated with the diode itself. These normalized slope parameters are used to relate the actual circuit and gain–bandwidth characteristics. The slope parameter of the external signal resonator is related to the negative slope parameter of the diode, and bounds on this ratio are given over which stable amplification is possible. A design table which gives the coupling ratio and slope parameter of the external signal resonator is derived by computer calculation. Experiments were made at 19 GHz. Positive-gain bandwidth was around 4.0 GHz, and flat bandwidth at 10-dB gain was 2.4 GHz. The ratio of these bandwidths coincided with the theory.

I. Introduction

THE idler resonant circuit of a parametric amplifier (paramp) is converted to a parallel resonant circuit (resonant at the signal frequency) with the negative conductance and negative susceptance slope parameter by the pumped nonlinear capacitance. The frequency response of this parallel resonant circuit can be compensated by an external parallel resonant circuit with a positive susceptance slope parameter. By this compensation, the flat bandwidth of the amplifier can be greatly increased. This procedure is a prerequisite to the design of wide-band paramps. If this is not undertaken, the full paramp bandwidth capability is not exploited.

DeJager [1] calculated the gain–bandwidth product of a paramp with a single-tuned signal and single-tuned idler circuit and the limiting gain–bandwidth product with single-tuned idler circuit. Connors [2] supplemented his work calculating maximally flat bandwidth with a single-tuned idler and a double-tuned signal circuit. In these papers, the loss in the signal circuit (series resistance of the diode: R_s) is neglected in order to get a simple gain–frequency expression. This can cause considerable error, especially when the signal circuit slope parameter is relatively low.

Getsinger [3] calculated the element value of the prototype signal circuit with prescribed gain and ripple. However, he also neglected the signal circuit loss R_s and did not consider the presence of the parallel resonant circuit with a negative slope parameter. If his method is applied to the paramps, the first element of the prototype circuit must be negative, which is not included in his table. Therefore, his design table is inapplicable to an optimum wide-band paramp design.

If signal circuit loss R_s is considered, there exists a limited frequency band over which amplifier gain is positive (in decibels). This frequency band corresponds to the band in which the real part of the pumped varactor impedance, presented to the circulator, is negative. This frequency limit does not exist if the signal circuit loss is neglected because, in this case, the real part of the impedance presented to the circulator is always negative.

In this paper, the frequency dependence of the gain is considered in terms of this frequency band. The idler and signal circuit slope parameters are normalized by the slope parameters attributable to the diode itself. These normalized slope parameters are used to derive relations between actual circuit and gain–frequency characteristics. Design parameters with prescribed gain and ripple are calculated on the computer and tabulated to serve as a design aid.

II. Definition of Normalized Signal and Idler Resonant Circuit Slope Parameters

The pumped nonlinear elastance (inverse of capacitance) of the diode can be expanded as

$$S(t) = S_0 + 2\,S_1 \cos(\omega_p t + \varphi_1) + 2S_2 \cos(2\omega_p t + \varphi_2) + \cdots \quad (1)$$

where ω_p is the angular frequency of pumping and S_0, S_1, S_2, $\cdots$, are the elastance coefficients, assumed to be real.

The dynamic Q of the varactor diode at the signal and idler frequency is defined as follows [6]:

$$\tilde{Q}_1 = S_1/\omega_{10}R_s \quad (2)$$

$$\tilde{Q}_2 = S_1/\omega_{20}R_s \quad (3)$$

where

- ω_{10} center of the signal frequency ω_1;
- ω_{20} center of the idler frequency ω_2;
- R_s series resistance of the diode.

Also, the modulation factor is defined as

$$\gamma = S_1/S_0. \quad (4)$$

The signal and idler resonant circuits, which are coupled by the pumped varactor elastance, can be depicted as in Fig. 1. The mean elastance S_0, lead inductance L_s, and series resistance R_s of the diode appear as common elements in the signal and idler circuits.

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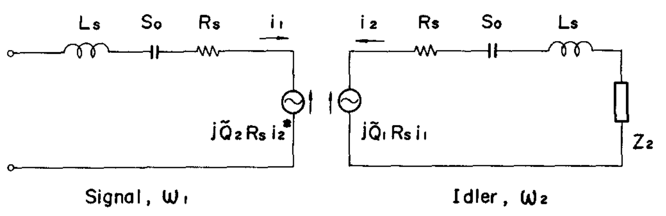

Fig. 1. Two frequency paramp equivalent circuit (Z_2 is the external idler circuit impedance, including package capacitance).

The frequency response of the resonant circuit near the resonant frequency can be conveniently represented by its slope parameter [4]. If the reactance of the series resonant circuit is $X(\omega)$, the reactance slope parameter at the resonant frequency ω_0 is represented as

$$x = \frac{\omega_0}{2} \frac{dX}{d\omega} \bigg|_{\omega=\omega_0} \quad \text{ohm.} \tag{5}$$

Also, if the susceptance of the parallel resonant circuit is $B(\omega)$, the susceptance slope parameter at the resonant frequency ω_0 is represented as

$$b = \frac{\omega_0}{2} \frac{dB}{d\omega} \bigg|_{\omega=\omega_0} \quad \text{mho.} \tag{6}$$

The signal and idler resonant circuits of the actual paramps are each series resonant circuit as shown in Fig. 1. Hence we represent the reactance slope parameter of the signal and idler resonant circuit by x_1, x_2, and define the slope parameters attributable to the diode itself as

$$x_{10} = \omega_{10} L_s \tag{7}$$

$$x_{20} = \omega_{20} L_s. \tag{8}$$

Here, we normalize the slope parameters x_1, x_2 x_{10}, x_{20} as follows:

$$a_1 = x_1 / x_{10} \tag{9}$$

$$a_2 = x_2 / x_{20} \tag{10}$$

a_1 and a_2 are always greater than or equal to 1, because L_s is included in both resonant circuits.

In the signal frequency circuit, the contribution of the idler resonant circuit is represented by the parallel resonant circuit with negative conductance $(-1/R_a)$ and negative susceptance slope parameter $(-b_a)$, as shown in Fig. 2. R_a and b_a are given as

$$R_a = \tilde{Q}_1 \tilde{Q}_2 R_s \tag{11}$$

$$b_a = \frac{a_2 x_{10}}{R_a R_s}. \tag{12}$$

III. Relative Bandwidth for Positive Amplifier Gain

Fig. 2 shows the equivalent circuit of the paramps with external signal resonator (b_3). Gain of this amplifier is positive if the real part of the impedance, seen from the circulator (the right-hand side of ①-①) is negative. This condition is determined by the right-hand side of

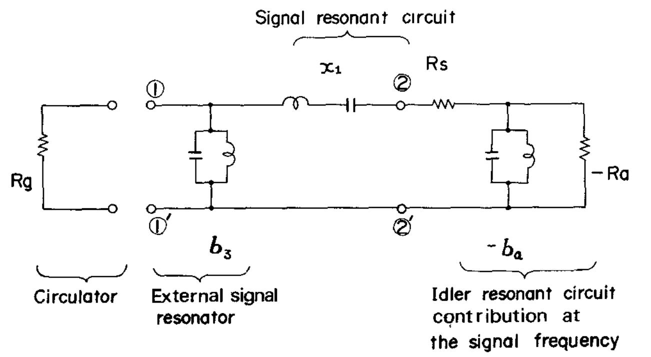

Fig. 2. Signal frequency equivalent circuit of wide-band paramp.

②-② and is represented as follows:

$$\text{Re}\left\{R_s - \frac{1}{R_a^{-1} + jb_a\Omega}\right\} \leq 0 \tag{13}$$

where

$$\Omega = \frac{\omega_1}{\omega_{10}} - \frac{\omega_{10}}{\omega_1} \approx \frac{2\Delta\omega_1}{\omega_{10}}, \qquad \Delta\omega_1 = \omega_1 - \omega_{10}. \tag{14}$$

The limit on the relative bandwidth Ω with positive gain is given as

$$0 < \Omega < \Omega_0, \qquad \Omega_0 = \frac{R_s}{a_2 x_{10}} (\tilde{Q}_1 \tilde{Q}_2 - 1)^{1/2}. \tag{15}$$

It must be emphasized that this relative bandwidth Ω_0 is independent of the midband amplifier gain and slope parameter of the external signal resonator (b_3). This can be considered as the upper bounds on relative bandwidth attainable by the paramp.

IV. Practical Signal and Idler Circuit Configurations

The diode and the circuit must be resonant at the signal and the idler frequency. To realize these resonances, it is practical to use the self-resonance of the diode. Here we introduce two simplified configurations which are applicable to lower and upper microwave frequency paramps with both single and balanced diode configurations.

Case A

The series resonance of the diode is used as the signal frequency resonance, and resonance, including the package capacitance and the external circuit, is used as idler resonance. This is depicted in Fig. 3, case A. This case is applicable to the upper microwave to lower millimeter-wave paramps with a single diode.

Case B

The series resonance of the diode is used as the idler resonance. To form the signal resonance, inductance L_a is added in the signal circuit only, to lower the resonant frequency of the diode. This case is applicable to lower microwave paramps and balanced diode paramps. This is depicted in Fig. 3, case B.

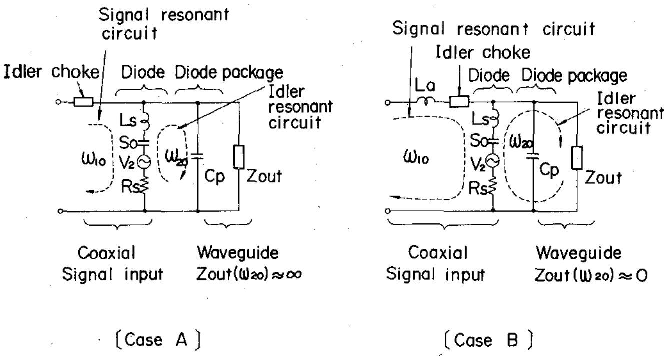

Fig. 3. Signal and idler resonant circuit configurations. V_2 is the idler voltage; Z_{out} is the impedance of the external idler circuit; C_p is the package capacitance; and L_a is the inductance added in the signal resonant circuit.

TABLE I
IDLER AND SIGNAL CIRCUIT CONFIGURATIONS (CF. FIG. 3)

	Resonant circuit	Inductance	Capacitance	Resonant Freq	Value of a_1 & a_2 [2]	Value of Ω_0 [3]
Case A	Signal	L_s	S_0^{-1} [1]	$(L_s/S_0)^{-\frac{1}{2}}$	$a_1 = 1$	$\frac{1}{a_2}\gamma\sqrt{\frac{w_{10}}{w_{20}}}$
	Idler	L_s	$(S_0+C_p^{-1})^{-1}$	$\{L_s/(S_0+C_p^{-1})\}^{-\frac{1}{2}}$	$a_2 = 1$	
Case B	Signal	L_s+L_a	S_0^{-1} [1]	$\{(L_s+L_a)/S_0\}^{-\frac{1}{2}}$	$a_1 = 1+L_a/L_s$	$\frac{1}{a_2}(1+\frac{L_a}{L_s})\gamma\sqrt{\frac{w_{10}}{w_{20}}}$
	Idler	L_s	S_0^{-1}	$(L_s/S_0)^{-\frac{1}{2}}$	$a_2 = 1$	

Note: Case A is applicable to upper microwave to lower millimeter-wave single-diode paramps. Case B is applicable to lower microwave and balanced diode paramps.
[1] Effect of C_p and Z_{out} is neglected.
[2] Contribution of the slope parameter of Z_{out} is neglected.
[3] Ω_0 is the relative bandwidth with positive gain.

If $\tilde{Q}_1\tilde{Q}_2 \gg 1$, which is satisfied by almost every paramps, (15) can be written as

$$\Omega_0 = \frac{R_s}{a_2 x_{10}} (\tilde{Q}_1\tilde{Q}_2)^{1/2}. \tag{16}$$

In case A, the signal resonant frequency is given by $\omega_{10} = (L_s/S_0)^{-1/2}$, therefore, using (2)–(4) and (7), (16) can be written as

$$\Omega_0 = \frac{\gamma}{a_2}\left(\frac{\omega_{10}}{\omega_{20}}\right)^{1/2}. \tag{17}$$

In case B, the signal resonant frequency is given by $\omega_{10} = \{(L_s + L_a)/S_0\}^{-1/2}$, therefore, with the same calculation, (16) can be written as

$$\Omega_0 = \frac{\gamma}{a_2}\left(1 + \frac{L_a}{L_s}\right)\left(\frac{\omega_{10}}{\omega_{20}}\right)^{1/2}. \tag{18}$$

These are the important simple relations which give the upper bounds on relative bandwidth of the paramps. In both cases, Ω_0 is proportional to γ. This means that, to achieve wide-band amplification, it is necessary to use a diode with which large γ is attainable.

Ω_0 of case B is larger than Ω_0 of case A by $1 + (L_a/L_s)$. This is confirmed by the fact that, for the balanced-type paramps, larger bandwidth is reported [5], [6]. These results are tabulated in Table I. The configurations and calculations in this section are first-order simplifications, which, however, provide the insight to the calculations stated hereafter. Simplification applied in this section does not affect the consistency of subsequent results.

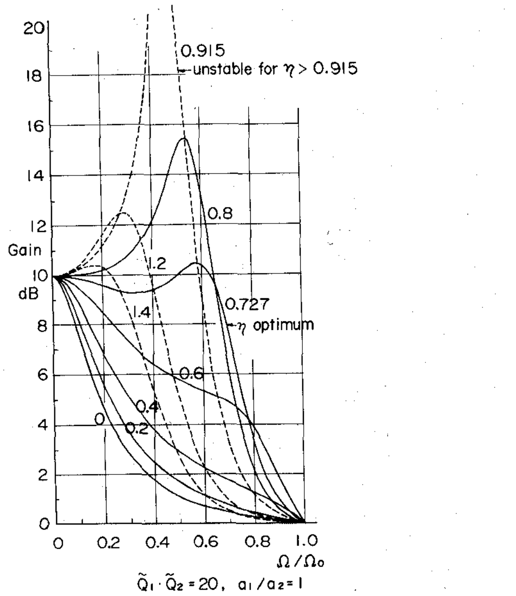

Fig. 4. Theoretical gain–frequency response varying the ratio of external signal resonator slope parameter to the negative slope parameter of the pumped diode. Dotted curve indicates unstable gain characteristic.

V. Paramp Gain Response with External Signal Resonator

As shown in Fig. 2, the pumped varactor presents a negative susceptance signal circuit slope parameter $-b_a$, which yields a narrow single-tuned gain response. If an additional parallel resonator with positive susceptance slope parameter b_3 is added, as shown in Fig. 2, $-b_a$ and b_3 will cancel each other and broad banding can be attained. Therefore b_3 is related to b_a as

$$\eta = b_3/b_a. \tag{19}$$

The reflection coefficient at ①–①′ of Fig. 2 is represented as

$$\Gamma(s) = \frac{\alpha(1 + \tilde{a}s)(1 + s)(1 - \beta\eta s) - (1 - \beta\eta s) - \beta(1 + s)}{\alpha(1 + \tilde{a}s)(1 + s)(1 + \beta\eta s) - (1 + \beta\eta s) + \beta(1 + s)} \tag{20}$$

where

$$\alpha = R_s/R_a = 1/\tilde{Q}_1\tilde{Q}_2 \tag{21}$$

$$\beta = R_g/R_a \tag{22}$$

$$\tilde{a} = a_1/a_2 \tag{23}$$

$$s = ja_2 \frac{x_{10}}{R_s} \Omega. \tag{24}$$

As an example, gain $|\Gamma|^2$ is shown in Fig. 4 as a function of Ω/Ω_0 with $\tilde{Q}_1\tilde{Q}_2 = 20$, $a_1/a_2 = 1$, and varying η.

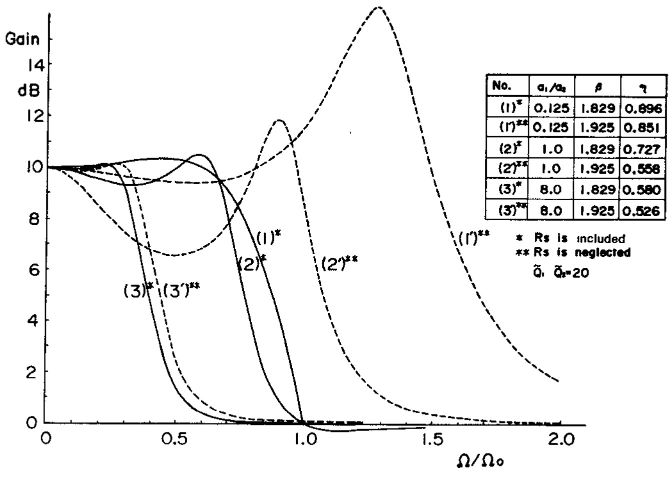

Fig. 5. Theoretical gain–frequency response with and without R_s in the signal circuit.

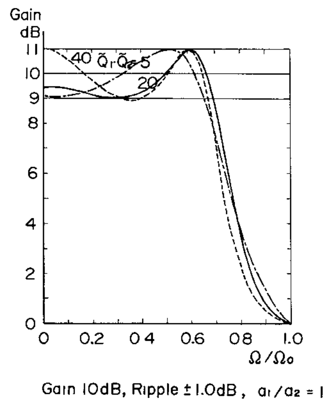

Fig. 6. Theoretical gain–frequency response for various $\tilde{Q}_1\tilde{Q}_2$ values.

$\eta = 0$ corresponds to the single-tuned case in which no external signal resonator is present. $\eta = 0.727$ is the optimum value with minimum deviation from 10-dB gain. For $\eta > 0.915$, the amplifier becomes unstable. Stability of paramps is considered in a later section.

VI. Effects of Neglecting Signal Circuit Loss R_s

In previous papers, signal circuit loss R_s is neglected on the condition that $R_s \ll R_a$. This approximation is valid for paramps with single-tuned signal circuits, but, in the case of paramps with double-tuned signal circuits, this is not so simple. It is easily understood that, if x_1 is relatively small and $R_s \approx 0$, almost infinite flat bandwidth can be obtained by choosing $b_3 \approx b_a$. This cannot happen for nonzero signal circuit R_s, because the bandwidth with positive gain is restricted by (15). In Fig. 5, gain frequency characteristics with and without R_s in the signal circuit are presented for three values of a_1/a_2. Optimum η in the figure is the value which give the minimum of the integral of $(|\Gamma|^2 - 10\text{ dB})$ within $0 \leqq \Omega/\Omega_0 \leqq 1$. It is seen therein that, when $a_1/a_2 \lesssim 1$, corresponding to relatively low signal circuit slope parameter, great error can arise in the gain–frequency characteristic by neglecting R_s. Since a typical value of a_1/a_2 is nearly unity, signal circuit loss must be considered to design for precise gain–frequency characteristics.

VII. Stability Considerations

For stable amplification, the denominator of (20) must not have roots in the right half-plane of complex frequency variable s. This can be satisfied if the denominator of (20) is a Hurwitz polynomial. In this case, the following condition must be satisfied:

$$\alpha + \beta - 1 > 0 \tag{25}$$

$$0 \leqq \eta < \frac{1}{\beta(\tilde{a}+1)(1-\alpha)} \Bigg[\frac{\beta}{2} \{\alpha(\tilde{a}+1)^2 + \beta\} + \left(\frac{\beta^2}{4} \{\alpha(\tilde{a}+1)^2 + \beta\}^2 - \tilde{a}\beta^2(\tilde{a}+1)(\alpha-1) \cdot \{\alpha(a+1) + \beta\} \right)^{1/2} \Bigg]. \tag{26}$$

If the gain at the center frequency is G_0, parameter β, which represents the coupling between the load and the negative resistance, has two solutions as

$$\beta_+ = (1-\alpha)\frac{1+G_0^{1/2}}{1-G_0^{1/2}} \qquad \beta_- = (1-\alpha)\frac{1-G_0^{1/2}}{1+G_0^{1/2}}. \tag{27}$$

Equation (25) indicates that in these two values of β, β_- is unstable.

The dotted curves in Fig. 4 correspond to the unstable η, which is determined by (26).

VIII. Bandwidth and Circuit Element Values for Prescribed Gain and Ripple

If R_s in the signal circuit is neglected, $(|\Gamma|^2 - 1)^{-1}$ can be represented by a simple polynomial in Ω. Connors [2] used this function to obtain an analytical solution with maximally flat response. But, if R_s is considered, the gain is given in terms of a rational function of Ω, for which an analytical solution with prescribed characteristics is impossible. In this paper, gain $(|\Gamma|^2)$ is derived from (20), which includes the effect of R_s, and values of β and η corresponding to a prescribed gain and ripple are calculated by computer techniques. The results of this calculation are shown in Table II, in which relative bandwidth over which the prescribed gain and ripple are obtained is represented by Ω_A, and Ω_A/Ω_0 is tabulated. It is seen therein that there are some combinations of parameters for which such solutions do not exist as indicated by blank entries for η and β. Figs. 6–8 exemplify the result of this calculation. Fig. 6 represents the solution corresponding to 10-dB gain, ±1-dB ripple, $a_1/a_2 = 1$, and three values of $\tilde{Q}_1 \cdot \tilde{Q}_2$. This figure and Table II show that a large value of $\tilde{Q}_1 \cdot \tilde{Q}_2$ does not increase the bandwidth ratio Ω_A/Ω_0 appreciably. Fig. 7 represents the solution corresponding to 10-dB gain, ±1-dB ripple, $\tilde{Q}_1 \cdot \tilde{Q}_2 = 20$, and four values of a_1/a_2. This figure and Table II show a

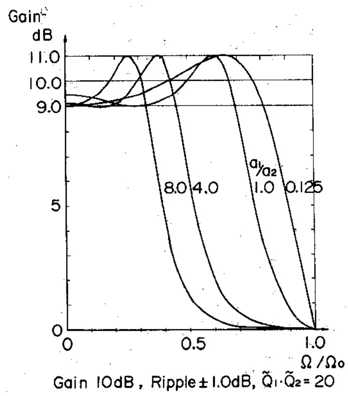

Fig. 7. Theoretical gain–frequency response for various a_1/a_2 values.

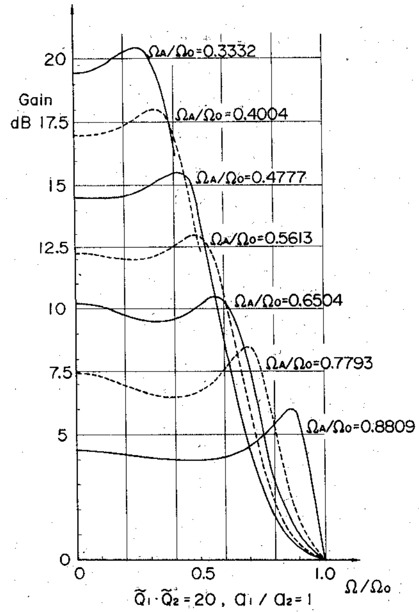

Fig. 8. Theoretical gain–frequency response for various midband gain levels. Prescribed ripple is ±0.5 dB (±1.0 dB for 5, 7.5 dB). Ω_A is the relative bandwidth which satisfies prescribed gain and ripple.

relatively large signal circuit Q, meaning large values of a_1/a_2, diminishes the bandwidth ratio Ω_A/Ω_0 of the paramps appreciably. Fig. 8 represents the solution corresponding to $\tilde{Q}_1 \cdot \tilde{Q}_2 = 20$, $a_1/a_2 = 1$, ± 0.5-dB ripple (± 1.0 dB for 5-, 7.5-dB gain), and variable gain.

Parameters a_1 and a_2 can be estimated from the equivalent signal and the idler resonant circuit. Relative bandwidth Ω_0 can be estimated by (17) or (18), or, more precisely, by experiment. Therefore, using Table II, the optimum slope parameter of the external signal resonator, optimum coupling to the diode, and the realizable relative bandwidth can be determined.

IX. Effect of the Idler Loading

Thus far resistance in the signal and idler circuit is assumed to be the same. However, there exists a possibility that the signal and the idler resistance have different values. Besides, it may be possible to add resistance intentionally in the idler circuit. When idler resistance is increased to ρR_s ($\rho \geqq 1$), the effects of this increasing can be taken into consideration simply by changing the parameters as

$$a_2 \rightarrow a_2/\rho \tag{28}$$

$$\tilde{Q}_1\tilde{Q}_2 \rightarrow \tilde{Q}_1\tilde{Q}_2/\rho. \tag{29}$$

In most cases, an increase of the idler circuit resistance also increases the positive-gain relative bandwidth Ω_0 as understood by (15). But Ω_0 take a maximum value when ρ is chosen as

$$\rho = \tfrac{1}{2}\tilde{Q}_1\tilde{Q}_2 \tag{30}$$

and maximum Ω_0 is given as

$$\Omega_{0\,\max} = \frac{\tilde{Q}_1\tilde{Q}_2R_s}{2a_2x_{10}}. \tag{31}$$

X. Experimental Results

This design theory was applied to a 19-GHz paramp (pump frequency 57 GHz) with the following parameters.

1) GaAs planer diffused diode was designed to resonate near 19 GHz. (Junction area is 10 $\mu\phi$, $C_{j0} = 0.15$ pF, $L_s = 0.4$ nH, $C_p = 0.08$ pF);

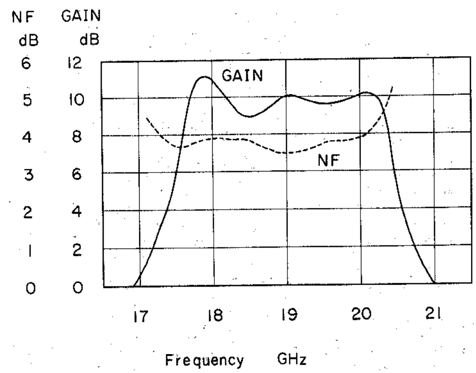

Fig. 9. Measured gain and noise figure of experimental 19-GHz paramp. The pumping frequency is 56.6 GHz, diode is 10 $\mu\phi$ diffused epitaxial planar GaAs diode. Noise figure and gain includes a circulator loss of 0.4 dB, a waveguide-coaxial transducer loss of 0.25 dB, and a signal circuit loss of 0.25 dB.

2) $a_1 \approx 1.5$, utilizing a new type of signal circuit adapted from quarterwave coupled BPF;

3) idler circuit was optimized to yield $a_2 \approx 2$;

4) increase of idler resistance is estimated as $\rho \approx 2$ from the experimental result of Ω_0;

5) $\tilde{Q}_1\tilde{Q}_2 \approx 20$ typically when $\tilde{Q}_2$ is estimated from $\tilde{Q}_1(\omega_{10}/\omega_{20})$.

The experimental data of positive-gain bandwidth were 4–4.5 GHz. Table II indicates that for $\tilde{Q}_1\tilde{Q}_2/\rho = 10$ and $a_2/\rho \approx 1$, and 10-dB gain (± 0.5-dB ripple), Ω_A/Ω_0 is around 0.60, yielding a predicted flat bandwidth of around 2.5 GHz. Fig. 9 depicts the measured gain and noise response of this paramp. The measured flat bandwidth of 2.4 GHz at 10-dB gain coincides with the above prediction. Therefore, utilizing the measured positive-gain bandwidth, the flat bandwidth at any desired gain can be correctly estimated, using Table II.

TABLE II
PARAMP DESIGN PARAMETERS β, η, AND OBTAINABLE Ω_A/Ω_0 FOR PRESCRIBED GAIN AND RIPPLE

			Ripple ±0.25 dB			±0.50 dB			±1.0 dB		
Gain	$\tilde{Q}_1 \cdot \tilde{Q}_2$	a_1/a_2''	β	η	Ω_A/Ω_0	β	η	Ω_A/Ω_0	β	η	Ω_A/Ω_0
7.5 dB	5	.125	2.0269	.9656	.6918	2.0917	.9977	.7754	2.2369	1.0391	.8512
		.250	2.0269	.9187	.7027	2.0917	.9453	.7785	2.2369	.9789	.8465
		.500	2.0269	.8453	.7066	2.0917	.8641	.7676	2.2369	.8906	.8262
		1.000	2.0269	.7516	.6676	2.0917	.7672	.7168	2.2369	.7875	.7668
		2.000	2.0269	.6687	.5645	2.0917	.6844	.6082	2.2369	.7055	.6551
		4.000	2.0269	.6297	.4246	2.0917	.6516	.4652	2.2369	.6797	.5090
		8.000	2.0269	.6687	.2895	2.0917	.7013	.3238	2.2369	.7437	.3598
	10	.125	2.2803	.8969	.7113	2.3531	.9148	.7887	2.5165	.9391	.8605
		.250	2.2803	.8531	.7449	2.3531	.8664	.8082	2.5165	.8836	.8652
		.500	2.2803	.7789	.7691	2.3531	.7867	.8090	2.5165	.7973	.8496
		1.000	2.2126	.6816	.7199	2.3088	.6867	.7527	2.4990	.6934	.7895
		2.000	2.1996	.5937	.6121	2.3016	.5984	.6449	2.4947	.6039	.6801
		4.000	2.2594	.5344	.4793	2.3531	.5414	.5098	2.5165	.5508	.5410
		8.000	2.2803	.5125	.3395	2.3531	.5266	.3676	2.5165	.5437	.3980
	20	.125	2.4070	.8852	.7395	2.4838	.8957	.8098	2.6564	.9098	.8730
		.250	2.4070	.8398	.7996	2.4838	.8457	.8418	2.6564	.8539	.8832
		.500				2.3355	.7621	.8246	2.5582	.7662	.8582
		1.000							2.3494	.6691	.7793
		2.000							2.3355	.5832	.6668
		4.000	2.2689	.5105	.5004	2.2561	.5188	.5121	2.4759	.5184	.5410
		8.000	2.3286	.4750	.3691	2.4370	.4781	.3910	2.6379	.4828	.4152
	40	.125	2.4703	.8910	.7902	2.5492	.8961	.8402	2.7263	.9035	.8895
		.250				2.4256	.8408	.8598	2.6521	.8445	.8926
		.500							2.2404	.7619	.8418
		1.000									
		2.000									
		4.000									
		8.000							2.4257	.4789	.4059
	80	.125	2.4567	.8910	.8348	2.5573	.9006	.8699	2.7612	.9042	.9043
		.250							2.3256	.8451	.8824
		.500									
		1.000									
		2.000									
		4.000									
		8.000									
10	5	.125	1.5720	1.0062	.6098	1.6059	1.0344	.6879	1.6799	1.0719	.7668
		.250	1.5720	.9687	.6090	1.6059	.9930	.6816	1.6799	1.0266	.7551
		.500	1.5720	.9125	.5949	1.6059	.9328	.6574	1.6799	.9609	.7215
		1.000	1.5720	.8453	.5371	1.6059	.8641	.5910	1.6799	.8906	.6465
		2.000	1.5720	.7969	.4348	1.6059	.8187	.4809	1.6799	.8484	.5293
		4.000	1.5720	.8031	.3145	1.6059	.8344	.3535	1.6799	.8766	.3941
		8.000	1.5720	.9250	.2098	1.6059	.9750	.2387	1.6799	1.0375	.2691
	10	.125	1.7685	.9180	.6238	1.8066	.9352	.7051	1.8899	.9574	.7816
		.250	1.7685	.8836	.6488	1.8066	.8961	.7145	1.8899	.9133	.7793
		.500	1.7685	.8266	.6590	1.8066	.8344	.7035	1.8899	.8465	.7520
		1.000	1.7611	.7531	.6043	1.8066	.7594	.6402	1.8899	.7687	.6785
		2.000	1.7648	.6883	.4973	1.8066	.6953	.5285	1.8899	.7062	.5629
		4.000	1.7685	.6516	.3691	1.8066	.6625	.3973	1,8899	.6797	.4309
		8.000	1.7685	.6562	.2488	1.8066	.6781	.2770	1.8899	.7062	.3066
	20	.125	1.8667	.9004	.6543	1.9070	.9102	.7270	1.9949	.9230	.7965
		.250	1.8667	.8648	.7066	1.9070	.8703	.7535	1.9949	.8785	.8020
		.500	1.8069	.8016	.6996	1.8707	.8047	.7340	1.9810	.8090	.7699
		1.000				1.7927	.7293	.6504	1.9132	.7312	.6832
		2.000				1.7998	.6629	.5387	1.9195	.6645	.5691
		4.000	1.8069	.6117	.3980	1.8707	.6141	.4207	1.9810	.6184	.4473
		8.000	1.8667	.5875	.2863	1.9070	.5953	.3059	1.9949	.6070	.3277
	40	.125	1.9159	.9035	.7020	1.9572	.9086	.7613	2.0474	.9154	.8168
		.250	1.8581	.8621	.8010	1.9199	.8645	.7816	2.0332	.8680	.8184
		.500							1.8544	.8010	.7590
		1.000									
		2.000							1.7403	.6672	.5457
		4.000							1.8508	.6137	.4355
		8.000				1.8693	.5762	.3145	1.9920	.5773	.3340
	80	.125	1.9283	.9105	.7590	1.9823	.9134	.8012	2.0736	.9160	.8363
		.250							1.8857	.8680	.8105
		.500									
		1.000									
		2.000									
		4.000									
		8.000							1.8044	.5887	.3199

XI. CONCLUSION

Taking resistance of the signal circuit R_s into account in a parametric amplifier, there exists a maximum bandwidth over which positive gain may be achieved, thus constituting the limiting bandwidth of the paramp. In this paper, flat bandwidth at any prescribed gain and ripple was related to this limiting bandwidth. Normalized signal and idler slope parameters were used to relate a gain–frequency characteristic to an actual circuit, thereby yielding a tabulation of design parameters including a source-to-diode coupling ratio and the slope parameter of an external signal resonator. An experimental paramp model, centered at 19 GHz, exhibited a positive-gain bandwidth of about 4 GHz and a 10-dB flat-gain bandwidth of about 2.4 GHz, which agrees closely with the theory.

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TABLE II (*Continued*)

Gain	$\tilde{Q}_1\tilde{Q}_2$	a_2/a_1	Ripple ±0.25 dB			±0.50 dB			±1.0 dB		
			β	η	Ω_A/Ω_0	β	η	Ω_A/Ω_0	β	η	Ω_A/Ω_0
12.5 dB	5	.125	1.3166	1.0531	.5277	1.3367	1.0797	.6027	1.3801	1.1156	.6809
		.250	1.3166	1.0250	.5191	1.3367	1.0484	.5887	1.3801	1.0820	.6621
		.500	1.3166	.9844	.4934	1.3367	1.0062	.5551	1.3801	1.0359	.6191
		1.000	1.3166	.9422	.4324	1.3367	.9656	.4863	1.3801	.9953	.5402
		2.000	1.3166	.9281	.3410	1.3367	.9547	.3840	1.3801	.9922	.4301
		4.000	1.3166	.9875	.2426	1.3367	1.0266	.2770	1.3801	1.0781	.3129
		8.000	1.3166	1.2062	.1590	1.3367	1.2687	.1840	1.3801	1.3437	.2090
	10	.125	1.4811	.9445	.5449	1.5038	.9602	.6207	1.5526	.9812	.6980
		.250	1.4811	.9172	.5551	1.5038	.9297	.6215	1.5526	.9461	.6887
		.500	1.4811	.8742	.5504	1.5038	.8836	.6012	1.5526	.8969	.6535
		1.000	1.4811	.8211	.4957	1.5038	.8297	.5340	1.5526	.8414	.5754
		2.000	1.4811	.7766	.3949	1.5038	.7875	.4285	1.5526	.8016	.4645
		4.000	1.4811	.7609	.2840	1.5038	.7781	.3145	1.5526	.8000	.3465
		8.000	1.4811	.8031	.1895	1.5038	.8312	.2145	1.5526	.8687	.2418
	20	.125	1.5634	.9180	.5684	1.5874	.9273	.6426	1.6388	.9398	.7152
		.250	1.5634	.8906	.6082	1.5874	.8965	.6605	1.6388	.9051	.7145
		.500	1.5542	.8430	.5584	1.5849	.8461	.6402	1.6388	.8516	.6777
		1.000	1.5231	.7852	.5324	1.5611	.7875	.5613	1.6281	.7910	.5934
		2.000	1.5318	.7344	.4316	1.5681	.7367	.4551	1.6334	.7410	.4832
		4.000	1.5588	.7000	.3230	1.5874	.7055	.3434	1.6388	.7148	.3676
		8.000	1.5634	.6922	.2199	1.5874	.7047	.2402	1.6388	.7219	.2629
	40	.125	1.6046	.9176	.6129	1.6291	.9225	.6754	1.6819	.9293	.7363
		.250	1.5904	.8861	.6345	1.6241	.8885	.6973	1.6819	.8920	.7355
		.500							1.6143	.8383	.6777
		1.000							1.5523	.7822	.5801
		2.000							1.5676	.7305	.4738
		4.000				1.5545	.6898	.3496	1.6316	.6902	.3707
		8.000	1.5858	.6621	.2434	1.6216	.6660	.2590	1.6819	.6715	.2754
	80	.125	1,6251	.9236	.6754	1.6500	.9256	.7145	1.7035	.9288	.7590
		.250							1.6300	.8901	.7348
		.500									
		1.000									
		2.000									
		4.000									
		8.000							1.6155	.6656	.2723
15	5	.125	1.1584	1.1031	.4512	1.1713	1.1297	.5238	1.1988	1.1633	.5973
		.250	1.1584	1.0844	.4410	1.1713	1.1078	.5051	1.1988	1.1406	.5746
		.500	1.1584	1.0578	.4090	1.1713	1.0812	.4676	1.1988	1.1125	.5277
		1.000	1.1584	1.0391	.3512	1.1713	1.0641	.4004	1.1988	1.0969	.4504
		2.000	1.1584	1.0578	.2715	1.1713	1.0891	.3105	1.1988	1.1312	.3520
		4.000	1.1584	1.1750	.1926	1.1713	1.2187	.2215	1.1988	1.2766	.2520
		8.000	1.1584	1.5000	.1262	1.1713	1.5656	.1449	1.1988	1.6531	.1660
	10	.125	1.3032	.9734	.4730	1.3177	.9875	.5418	1.3487	1.0074	.6168
		.250	1.3032	.9523	.4730	1.3177	.9648	.5363	1.3487	.9820	.6020
		.500	1.3032	.9203	.4559	1.3177	.9309	.5082	1.3487	.9453	.5613
		1.000	1.3032	.8836	.4020	1.3177	.8937	.4426	1.3487	.9082	.4855
		2.000	1.3032	.8578	.3145	1.3177	.8703	.3480	1.3487	.8891	.3848
		4.000	1.3032	.8656	.2230	1.3177	.8859	.2520	1.3487	.9141	.2824
		8.000	1.3032	.9500	.1488	1.3177	.9828	.1707	1.3487	1.0266	.1941
	20	.125	1.3756	.9375	.4949	1.3910	.9457	.5613	1.4236	.9578	.6340
		.250	1.3756	.9164	.5184	1.3910	.9223	.5707	1.4236	.9311	.6270
		.500	1.3756	.8809	.5137	1.3910	.8848	.5480	1.4236	.8910	.5879
		1.000	1.3697	.8375	.4504	1.3894	.8406	.4747	1.4236	.8461	.5082
		2.000	1.3727	.8000	.3566	1.3910	.8047	.3801	1.4236	.8117	.4059
		4.000	1.3756	.7797	.2574	1.3910	.7883	.2785	1.4236	.8000	.3027
		8.000	1.3756	.7875	.1707	1.3910	.8039	.1918	1.4236	.8266	.2137
	40	.125	1.4118	.9320	.5301	1.4276	.9367	.5910	1.4611	.9434	.6535
		.250	1.4118	.9086	.5746	1.4276	.9111	.6098	1.4611	.9148	.6496
		.500				1.3968	.8709	.5676	1.4456	.8723	.5996
		1.000							1.4196	.8271	.5098
		2.000				1.3781	.7875	.3871	1.4292	.7877	.4105
		4.000	1.3880	.7559	.2793	1.4134	.7578	.2957	1.4558	.7609	.3145
		8.000	1.4118	.7414	.1957	1.4276	.7480	.2105	1.4611	.9203	.2262
	80	.125	1.4299	.9361	.5871	1.4459	.9382	.6301	1.4798	.9414	.6770
		.250				1.4073	.9098	.6254	1.4591	.9107	.6590
		.500							1.3724	.8727	.5816
		1.000									
		2.000									
		4.000							1.4015	.7586	.3066
		8.000				1.4132	.7348	.2176	1.4625	.7359	.2316

Ω_A is the obtainable relative bandwidth when gain and ripple is prescribed.
Ω_0 is the positive gain relative bandwidth.

References

[1] J. T. DeJager, "Maximum bandwidth performance of a non-degenerate parametric amplifier with single-tuned idler circuit," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-12, pp. 459–467, July 1964.

[2] W. P. Connors, "Maximally flat bandwidth of a nondegenerate parametric amplifier with double tuned signal circuit and single tuned idler circuit," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 251–252, Mar. 1965.

[3] W. J. Getsinger, "Prototype for use in broad banding reflection amplifiers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-11, pp. 486–497, Nov. 1963.

[4] G. L. Matthaei *et al.*, *Microwave Filters, Impedance-Matching Networks and Coupling Structures.* New York: McGraw-Hill, 1964.

[5] D. Neuf, R. Holifield, and E. Barnell, "S-band nondegenerative parametric amplifier with 1000-Mc instantaneous bandwidth," *Proc. IEEE*, vol. 51, pp. 1365–1366, Oct. 1963.

[6] H. A. Watson, *Microwave Semiconductor Devices and Their Circuit Applications.* New York: McGraw-Hill, 1969.

[7] Kim and Meadows, *Modern Network Analysis.* New York: 1971.